**Lecture 3.**

**Sequences of real numbers. Limit of a sequences.**

**Monotonic sequences. Cauchy`s Convergence Criterion.**

**Sequences of Real Numbers**

An infinite sequence (more briefly, a sequence) of real numbers is a real-valued function defined on a set of integers $\left\{n|n\geq k\right\}. $ We call the values of the function the terms of the sequence. We denote a sequence by listing its terms in order; thus,

 $\left\{s\_{n}\right\}\_{k}^{\infty }=\left\{s\_{k},s\_{k+1},…\right\}.$ (1)

The real number $s\_{n}$ is the nth term of the sequence. Usually we are interested only in the terms of a sequence and the order in which they appear, but not in the particular value of k in (1). Therefore, we regard the sequences

$$\left\{\frac{1}{n-2}\right\}\_{3}^{\infty } and \left\{\frac{1}{n}\right\}\_{1}^{\infty }$$

as identical.

**Definition.** If for every positive number $ε$ and $∀n>n\_{ε}$ there is $n\_{ε}$ such that , then the sequence  is called *convergent sequence* and a number ***а*** is called the *limit of sequence* and we write ***.***

A sequence that does not converge diverges, or is divergent.

**Uniqueness of the Limit**

**Theorem.** The limit of a convergent sequence is unique.

**Monotonic Sequence**

**Definition.** The sequence is said to be increasing if $x\_{n}<x\_{n+1}$, nondecreasing if $x\_{n}\leq x\_{n+1}$, decreasing if $x\_{n}>x\_{n+1}$, nonincreasing if $x\_{n}\geq x\_{n+1}$. A sequence that satisfies any of these conditions is called *monotonic*.

If the sequence  has only positive numbers, i.e.,   > 0 then monotonicity of sequence can be shown by conditions: the sequence is increasing if  > 1, the sequence is nondecreasing if   1, the sequence is decreasing if  < 1, and the sequence is nonincreasing if

   1.

**Theorem.** A convergent sequence is bounded.

**Definition.** A sequence $\left\{s\_{n}\right\}$ is nondecreasing if $s\_{n}\geq s\_{n-1}$ for all n, or nonincreasing if $s\_{n}\leq s\_{n-1}$ for all n. A monotonic sequence that is either nonincreasing or nondecreasing. If $s\_{n}>s\_{n-1 }$for all n, then $\left\{s\_{n}\right\}$ is increasing, while if $s\_{n}<s\_{n-1 }$for all n, $\left\{s\_{n}\right\}$ is decreasing.

**Theorem.**

1. if $s\_{n}$ is nondecreasing, then $\lim\_{n\to \infty }s\_{n}=sup\left\{s\_{n}\right\}.$
2. if $s\_{n}$ is nonincreasing, then $\lim\_{n\to \infty }s\_{n}=inf\left\{s\_{n}\right\}.$

**Cauchy’s Convergence Criterion**

**Theorem** (Cauchy’s Convergence Criterion). The sequence  converges  if it is *fundamental*  * *>0  : ,   **  <**.